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TRANSIENT MASS TRANSPORT BETWEEN A FINITE VOLUME OF HOMOGENIZED FLUID AND A SPHERE WITH FINITE INTERFACIAL TRANSPORT COEFFICIENTS

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NOMENCLATURE

b, root of equations (7) or (9);
C, solute concentration (or temperature) in sphere;
D, diffusion coefficient (or thermal diffusivity)
F, solute concentration (or temperature) in fluid;
k, mass transfer coefficient (or heat transfer coefficient/volumetric heat capacity of sphere);
r, radial distance from sphere center;

R, radius of solid sphere;
t, real time;
V, real fluid volume [for heat transfer, V = (real fluid volume) (volumetric heat capacity ratio of fluid to sphere)];
β, dimensionless volume, $V/[(4/3)\pi R^3]$;
ε, dimensionless interfacial transport coefficient, kR/D
τ, dimensionless time, Dt/R^2;
ξ, dimensionless radial distance, r/R.

Superscript
* dimensionless concentrations, $C^* = (C_0 - C)/(C_0 - F_0)$ and $F^* = (C_0 - F)/(C_0 - F_0)$.

Subscripts
o, at $t = 0$.

INTRODUCTION

TRANSIENT diffusion of mass or heat from a sphere to its surrounding liquid is of interest to leaching solids, gas absorption, solvent extraction, heat-treating of metals and other practical operations. Analytic mathematical solutions for a very large fluid volume per sphere with or without interfacial resistance and that for a finite fluid volume per sphere having no interfacial resistance are available [1]. This short article provides an analytical solution to describe the transient phenomenon of a sphere in a finite volume of homogenized fluid with a finite interfacial resistance. Some calculation results are also included.

THEORY

The transport of mass in a sphere is described by equation

Table 1. Calculated values of $(1 - F^*)(\beta + 1)$

ε	β	$\tau = 0{\cdot}02$	0·04	0·08	0·16	0·20
1000	0·5	0·7238	0·8340	0·9229	0·9792	0·9889
	1	0·6157	0·7482	0·8700	0·9588	0·9764
	2	0·5325	0·6732	0·8160	0·9333	0·9592
	4	0·4796	0·6213	0·7743	0·9103	0·9428
	10	0·4432	0·5837	0·7419	0·8905	0·9280
	1000	0·4169	0·5556	0·7164	0·8738	0·9151
500	0·5	0·7224	0·8333	0·9226	0·9791	0·9889
	1	0·6138	0·7471	0·8694	0·9586	0·9763
	2	0·5305	0·6717	0·8151	0·9329	0·9590
	4	0·4775	0·6197	0·7732	0·9097	0·9424
	10	0·4411	0·5819	0·7406	0·8898	0·9275
	1000	0·4148	0·5538	0·7151	0·8730	0·9145
100	0·5	0·7113	0·8275	0·9201	0·9784	0·9885
	1	0·5991	0·7376	0·8645	0·9568	0·9751
	2	0·5142	0·6599	0·8078	0·9296	0·9568
	4	0·4609	0·6066	0·7643	0·9052	0·9392
	10	0·4245	0·5683	0·7307	0·8843	0·9234
	1000	0·3984	0·5398	0·7045	0·8667	0·9097
50	0·5	0·6968	0·8198	0·9168	0·9774	0·9880
	1	0·5805	0·7256	0·8580	0·9544	0·9737
	2	0·4942	0·6450	0·7985	0·9253	0·9539
	4	0·4408	0·5904	0·7531	0·8994	0·9350
	10	0·4046	0·5515	0·7183	0·8772	0·9181
	1000	0·3790	0·5227	0·6912	0·8587	0·9035
10	0·5	0·5740	0·7474	0·8849	0·9680	0·9826
	1	0·4450	0·6242	0·7989	0·9309	0·9585
	2	0·3617	0·5310	0·7186	0·8852	0·9254
	4	0·3142	0·4731	0·6619	0·8465	0·8952
	10	0·2836	0·4338	0·6205	0·8149	0·8693
	1000	0·2625	0·4059	0·5896	0·7895	0·8478

(1) and material balance of solute in the homogenized fluid is described by equation (2). The initial condition is equation (3) and boundary conditions are equations (4) and (5).

$$\frac{\partial C}{\partial t} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right) \text{for } 0 \leqslant r \leqslant R \tag{1}$$

$$4\pi R^2 D \left.\frac{\partial C}{\partial r}\right|_R + V\frac{\mathrm{d}F}{\mathrm{d}t} = 0 \text{ for } r > R. \tag{2}$$

$$\text{At } t = 0, C = C_0 \quad \text{for } 0 \leqslant r \leqslant$$
$$F = F_0 \quad \text{for } r > R. \tag{3}$$

$$\text{At } r = 0, C = \text{finite} \tag{4}$$

$$\text{At } r = R, -D\frac{\partial C}{\partial r} = k(C - F). \tag{5}$$

For gas absorption and solvent extraction, they are restricted to those without convective transport within the sphere. If a partition constant is defined as the equilibrium concentration ratio of solute in the sphere phase to that in the surrounding fluid phase, F equals the product of the partition constant and the real solute concentration in the fluid phase. Also, V would be an equivalent fluid volume, i.e. the real fluid volume divided by the partition constant.

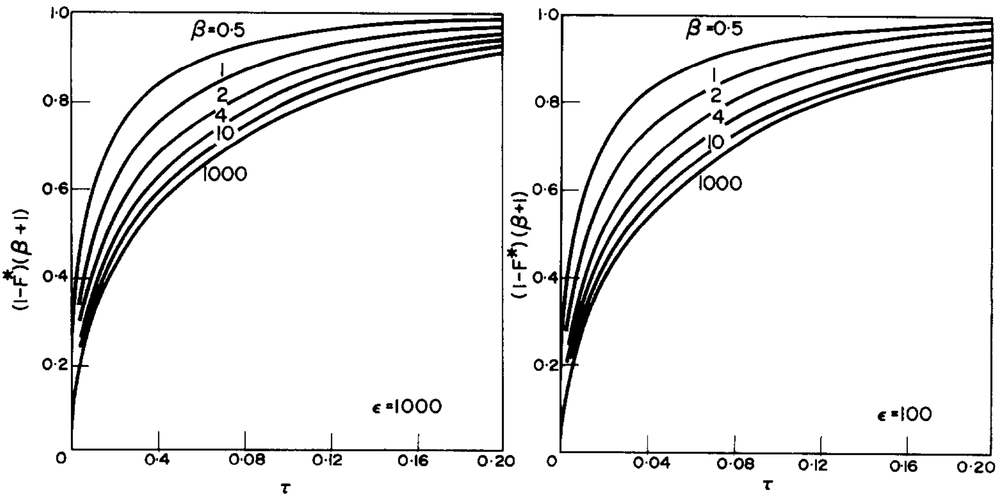
This set of coupled partial differential equations and their initial and boundary conditions can be simplified to the following in terms of dimensionless parameters:

$$\frac{\partial C^*}{\partial \tau} = \frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2\frac{\partial C^*}{\partial \xi}\right) \quad \text{for } 0 \leqslant \xi \leqslant 1 \tag{1A}$$

$$\frac{3}{\beta}\left.\frac{\partial C^*}{\partial \xi}\right|_{\xi=1} + \frac{\mathrm{d}F^*}{\mathrm{d}\tau} = 0 \quad \text{for } \xi > 1. \tag{2A}$$

Table 2. Five roots of equation (7)

ε	β	b_1	b_2	b_3	b_4	b_5
1000	0·5	3·9711	6·9354	9·9356	12·9761	16·0447
	1	3·7248	6·6767	9·7074	12·7812	15·8775
	2	3·5036	6·4968	9·5687	12·6708	15·7866
	4	3·3458	6·3918	9·4937	12·6130	15·7398
	10	3·2286	6·3240	9·4470	12·5776	15·7113
	1000	3·1394	6·2774	9·4157	12·5540	15·6924
500	0·5	3·9702	6·9321	9·9291	12·9662	16·0313
	1	3·7231	6·6719	9·6992	12·7695	15·8626
	2	3·5014	6·4911	9·5598	12·6586	15·7712
	4	3·3432	6·3858	9·4844	12·6006	15·7242
	10	3·2257	6·3178	9·4377	12·5651	15·6957
	1000	3·1363	6·2711	9·4063	12·5415	15·6767
100	0·5	3·9629	6·9051	9·8754	12·8836	15·9195
	1	3·7100	6·6329	9·6322	12·6749	15·7411
	2	3·4831	6·4457	9·4875	12·5602	15·6472
	4	3·3216	6·3376	9·4104	12·5011	15·5997
	10	3·2020	6·2683	9·3629	12·4653	15·5712
	1000	3·1111	6·2209	9·3311	12·4416	15·5523
50	0·5	3·9536	6·8699	9·8044	12·7740	15·7726
	1	3·6932	6·5828	9·5460	12·5542	15·5883
	2	3·4600	6·3881	9·3964	12·4376	15·4952
	4	3·2946	6·2771	9·3181	12·3787	15·4490
	10	3·1724	6·2066	9·2704	12·3435	15·4216
	1000	3·0798	6·1587	9·2387	12·3203	15·4036
10	0·5	3·8712	6·5327	9·1620	11·9338	14·8464
	1	3·5485	6·1534	8·8929	11·7800	14·7598
	2	3·2680	5·9361	8·7700	11·7133	14·7218
	4	3·0765	5·8258	8·7128	11·6825	14·7040
	10	2·9392	5·7603	8·6800	11·6647	14·6937
	1000	2·8374	5·7176	8·6589	11·6533	14·6870



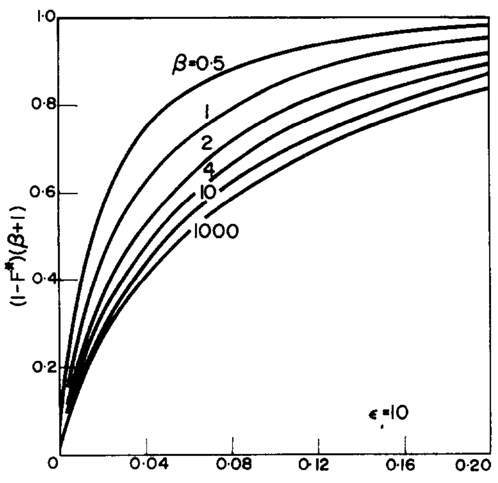


FIG. 1. Some transient concentrations of fluid as calculated from equations (6) and (7).

Table 3. *Summary of data*

Particles characteristics	
equivalent radius	= 0·2207 cm
porosity	= 0·472
Leaching experiment variables	
number of particles	= 322
volume of liquid	= 1700 ml
speed of stirrer	= 500 rpm

$$\text{At } \tau = 0,\ C^* = 0 \quad \text{for } 0 \leqslant \xi \leqslant 1$$
$$F^* = 1 \quad \text{for } \xi > 1. \tag{3A}$$

$$\text{At } \xi = 0,\ C^* = \text{finite.} \tag{4A}$$

$$\text{At } \xi = 1,\ \frac{\partial C^*}{\partial \xi} = \varepsilon(C^* - F^*). \tag{5A}$$

These equations can be solved by using the Laplace transform and Heaviside inversion [2]. The analytical solution is found to be:

$$F^* = \frac{\beta}{\beta + 1} + 6\beta \sum_{j=l}^{\infty} \frac{\exp(-b_j^2\tau)}{(\beta/\varepsilon)^2 b_j^4 + \{\beta^2 - (\beta/\varepsilon)[\beta + 6]\}b_j^2 + 9(\beta + 1)} \tag{6}$$

where b_j is the jth root of:

$$\tan b = \frac{(3\varepsilon - \beta b^2)b}{3\varepsilon + \beta(\varepsilon - 1)b^2}. \tag{7}$$

The case of no interfacial resistance corresponds to ε approaching infinity. Under this condition, equations (6) and (7) approach (8) and (9) which are available solutions [1].

$$F^* = \frac{\beta}{\beta + 1} + 6\beta \sum_{j=l}^{\infty} \frac{\exp(-b_j^2\tau)}{\beta^2 b_j^2 + 9(\beta + 1)} \tag{8}$$

where b_j is the jth root of:

$$\tan b = \frac{3b}{3 + \beta b^2}. \tag{9}$$

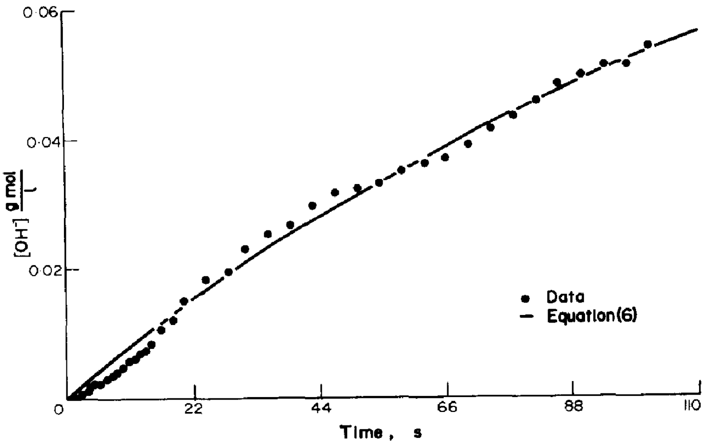


FIG. 2. Modeling experimental data by using equation (6).

CALCULATED RESULTS

Equations (7) and (8) are solved by using an IBM 360 computer. The Newton-Raphson method with the modification of imposing quadrant limits was used for solving equation (7). To cover the useful range of ε from 10 to 5000. five sets of computation results are summarized in Tables 1 and 2 for $\varepsilon = 10, 50, 100, 500$ and 1000. Results in Table 1 used b_1 up to b_{10}. However, only b_1 to b_5 are shown in Table 2 since the higher order terms have very little effect. The results of $\varepsilon = 1000$ are very close to the values from equations (8) and (9). Similarly, $\beta = 1000$ corresponds to results which are very close to $\beta = \infty$.

The calculation results and equations (6) and (7) should be useful to the design of equipment and operations in the mentioned subject areas. The results are also plotted in Fig. 1. The variables are grouped similar to that of Carslaw and Jaeger [1] for purpose of comparison. Curves of $\varepsilon = 1000$ coincide with those of Fig. 30 of that reference.

MODELING OF EXPERIMENTAL DATA

Equation (6) is useful to model data obtained recently [3]. The experiment consists of dumping a batch of porous cylinders soaked with dilute NaOH solution into a 5 l. Brunswick fermenter with distilled water in agitation. The acidity of the liquid phase is monitered by a pH recorder on a strip chart. Table 3 displays operation data.

These data points showed oscillation due to insufficient agitator speed which, at a high rate, would cause breakage of particles. At the experimental speed, only part of the particles are suspended. Therefore, equation (6) can be considered only as a model.

A non-linear optimization was used to seek the parameters D and k in equation (6) to correlate data points. The results are on Fig. 2. $D = 1{\cdot}19 \times 10^{-4}$ cm^2/s and $k = 5{\cdot}40 \times 10^{-4}$ cm/s were obtained.

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